

**TEACHING AND TESTING FOR
STATISITICAL THINKING**

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I Introduction:

I want to start this presentation with the case study of a “social studies” teacher who was very bored with his teaching job. He was not challenged any more and he thought that his students were incapable of analyzing and evaluating the important and critical episodes in world history. He thought that one solution to this dilemma would be to write exam questions that required his students to engage in critical and upper level thinking. He spent quite a long time writing such an exam and was happy about his accomplishment. However, after he administered the exam, more than 90% of the students could not answer the questions he had designed. After some deliberation he came to the conclusion that he was not teaching for thinking. In other words:

“HE WAS TESTING FOR SOMETHING that HE DID NOT TEACH”

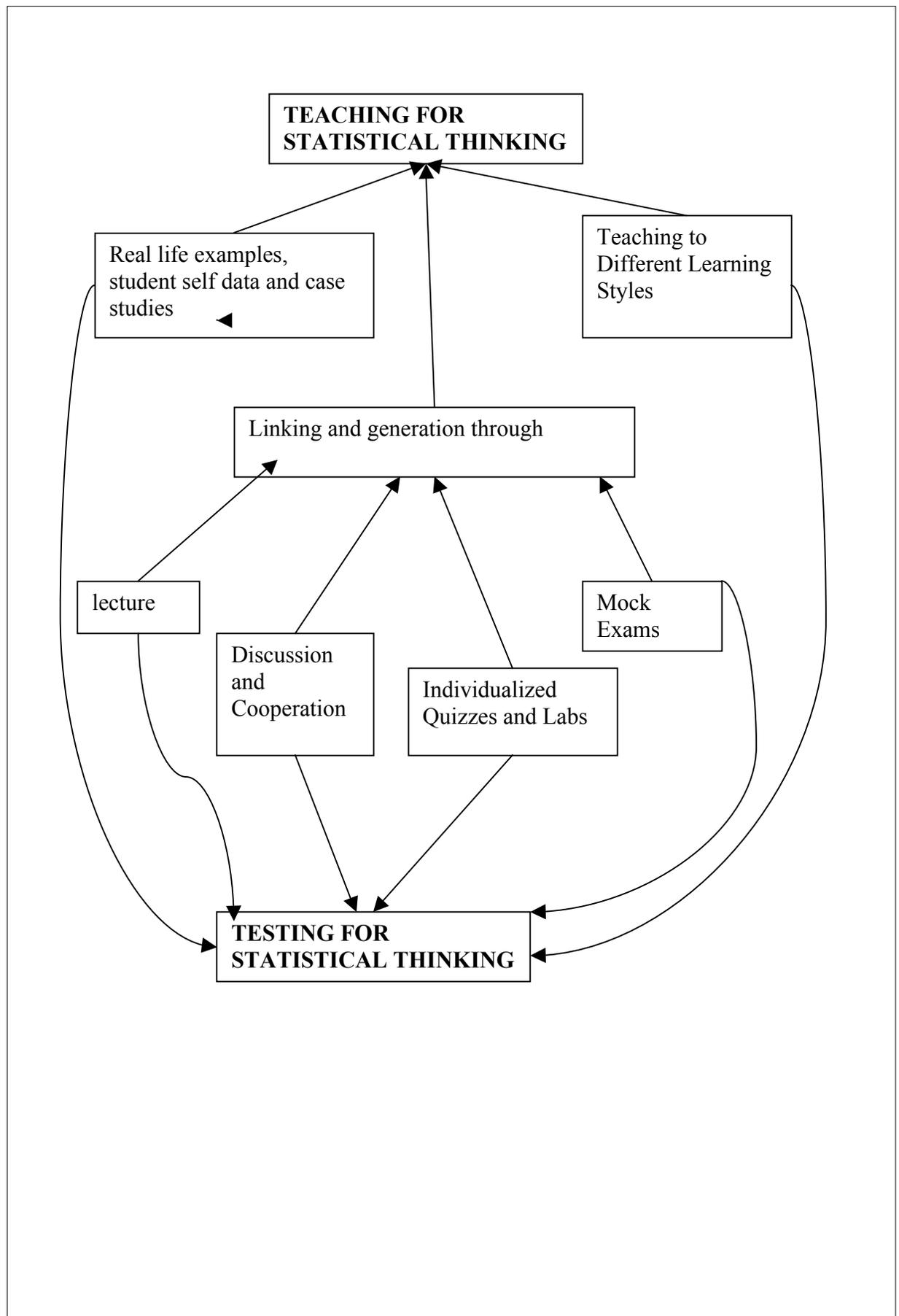
Conclusion:

Teaching for thinking and testing for thinking is a two-way and not a one-way street



II A MODEL FOR TEACING AND STETING OF STATISTICAL THINKING

**IN THE FOLLOWING I WILL
PROPOSE A MODEL TO TEACH
AND TEST FOR STATISTICAL
THINKING**



III FOUNDATIONS OF THE MODEL ON TEACHING AND TESTING FOR STATISTICAL THINKING

III.1 THEORETICAL FOUNDATION OF THE MODEL:

JEROME BRUNER: Learning is an active process in which students construct new ideas or concepts based on their current knowledge. Learners are encouraged to discover facts and relationships for themselves. You can teach anything to anybody if you teach it in their language.

JEAN PIAGET: Growth of knowledge IS a progressive process in which the learner plays an active role. Learning is a process of equilibration between assimilation and accommodation and it results from cognitive conflict.

MERLIN WITTROC: Students should be encouraged to generate their own knowledge by going from the old to the new, the simple to the more complex, and by creating links between the different components.

**CASE BASED APPROACH TO TEACHING AND LEARNING
COOPERATIVE GROUPS**

III.2 THE PRACTICAL FOUNDATIONS OF THE PROPOSED MODLE

III.2.1 WHAT HAS HELPED ME IN TEACHING FOR THINKING

Teaching a concept with different methods:

Equations, plots, examples, verbal explanations, tables, computer printout, or whatever source that I have found useful

Modeling statistical thinking and what I expect the students to do: State a problem, identify the variables, decide what statistical method can be used to solve the problem, conduct the analysis, interpret the results within the context of the problem.

Creating an active role for the students during the lecture by minimizing lecturing on my part and maximizing participation by the students

Getting to know the students' misconceptions by listening to them while they work with a neighbor, analyzing their answers to my questions during lecture, and analysis of incorrect answers to exam questions.

Modeling statistical thinking and upper level thinking through mock exams an on-line quizzes.

The students work on these mock exams or on-line quizzes at home and later they discuss the answers with a neighbor or a group during lecture or discussion.

III.2.2 WHAT HAS HELPED THE STUDENTS ENGAGE IN STATISTICAL THINKING

I asked Stat 10 students to respond to the following question:

Of the things that we have done throughout the quarter, what has helped you as an individual to think rather than to memorize or do calculations blindly?

There were a total of 43 written responses to this open-ended question. These responses fit in four major categories:

- a) 11/43 or 25.6% indicated that they engaged in statistical thinking through linking the material in the lecture.
- b) 12/43 or 27.9% indicated that they engaged in statistical thinking through the mock exams.
- c) 8/43 or 18.6% of the comments related to engaging in statistical thinking through interaction with their neighbor during lecture.
- d) 3/43 or 6.9% of the comments related to engaging in statistical thinking through exams
- e) 9/43 or 20.9% of the comments related to engaging in statistical thinking through case studies, real world examples, relevance of context, and paying attention to different learning style

IV Summary of student comments

(4) Shows that they thought they engaged in statistical thinking to a large extent.

(3) Indicates that they though they engaged in statistical thinking to some extent.

- 1) (4) Constantly relating the new concepts to the ones we have already covered; it helps me understand the new and old better than I would otherwise.
- 2) (4) The cases and the interactions.
- 3) (4) The case studies that help explain class information are very helpful in understanding rather than memorizing. Mock quizzes and midterm reviews have helped us not memorize formulas as much as understand them.
- 4) (4) Mock quizzes made me think and not the homeworks.
- 5) (3) mock quizzes and review midterm.
- 6) (4) mock quizzes and explanation of different topics in this quarter
- 7) (3) I learned the most from the chapter on setting up experiments.
- 8) (4) Going through the theory behind each topic as well as the application of it in the real world.
- 9) (4) I like how I first think and then memorize.

- 10) (5) Your success to teach for every learning style has helped me understand the material in many different ways, using real life examples that has relevance to people within my age group.
- 11) (3) Talking to a neighbor and listening to a peer describe how to think about a problem. Your step by step approach to solving a statistic problem has also been very helpful.
- 12) (3) open book and open note encourages you to understand the concept rather than memorize the formulas, talking to a neighbour
- 13) (3) Having to try to explain to my neighbor during class the actual meaning of something has shown me what I truly understand and do not and has shown me what I need to study further
- 14) (4) The mock quiz and the review midterm have helped me think, because I could see the thought process behind how to answer the questions, the lecture notes also help a lot
- 15) (4) Mock quizzes, how you connect concepts from previous lecture, summary offered at the end of the lecture
- 16) (4) Thinking about concepts and applying them to statistics, figuring out what happens if conditions change and how the change affects the formula.
- 17) (4) Your teaching style and how you emphasize that learning is more important than memorization

- 18) (4) Your interaction with the class, you talk with us and not at us. You actually try to help us understand and not just giving us the book.
- 19) (3) Learning the importance of the context of the situation has taught me to think about each specific problem more
- 20) (4) Structuring the exam on the principals of the material taught and asking us in class to interpret real life statistics
- 21) (4) Type of questions asked in the exam required me to think
- 22) (3) Application of stat instead of plugging numbers, mock quizzes
- 23) (3) Talking about the problem with the person next to you, discussion of mock quizzes in class
- 24) (3) The book, mock quizzes, homework, practice midterm
- 25) (4) Pictures and real life examples
- 26) (4) Connecting the old and the new
- 27) (4) Working out the problems in class have helped me to see if my approach is correct or not. Talking to my neighbor has also worked and they helped me answer the questions. When I do homework at home, I do not have anyone to check things with.

- 28) (4) The lectures have helped me think. I think it is hard to engage in thinking when you are required to memorize. I admire how you are trying to help us really think about what we are learning.
- 29) (4) Stopping and having us talk to our neighbor, giving time to ask questions, doing sample problems, instead of just going up to the board and writing problems for 50 minutes.
- 30) (4) Mock quizzes, interaction in the class, and the fact that we can all participate

V The role of technology (quiz tool) in teaching and testing for thinking

- Use weekly on-line quizzes as a mean of motivating students to play an active role in their own learning on an individual level. This is done by having the students spend 30-45 minutes to take the on-line quiz by themselves.
- Use the quiz tool as a mean of formative evaluation and allowing the students to revisit and discuss the concepts that the majority did not master. This is done by a group discussion of the test items that the majority got wrong.
- Use the quiz tool to allow the TA and the instructor to re-teach the concepts that the majority of the students missed.
- Use the percentage of wrong answers to test items provided by the quiz tool as a mean of pinpointing students' misconceptions.
- Use participation of the TA and instructor in student groups as a mean of learning how students think about statistical concepts and what is confusing to them

VI The types of questions that do not allow us to test for thinking

- True-false questions that have no real context and do not require the students to defend their answers.
- Questions that require the students to follow stepwise calculations blindly and do not require them to show any real understanding of the material.
- Multiple choice questions that ask for facts and memorized information.
- Multiple choice questions in which the student can get the right answer by the elimination of the wrong answer.

VII Types of questions that allow us to test for THINKING

- short answers questions that do not have a single right answer, are easy to grade, and require the students to think.
- Multiple choice questions that require the students to link, apply, analyze, synthesize, and evaluate the relevant information.
- Multiple choice questions the answer to which involves more than one component and can be given partial credit.
- Word problems that allow multiple interpretations and different ways of answering the same question.

VIII Characteristics of questions that help us test for thinking

In order to get the correct answer, the student should:

- Link information,
- See the connection between the different parts that make up the right answer,
- Have a very thorough and not superficial understanding of the concepts, equations, plots, calculations, or whatever that involves finding the right answer,
- Be able to see the big picture rather than unrelated pieces of information,
- See the details within the big picture as well as the connection between these details (webbing), and
- be able to see the concept from different angles.

IX THE CHARACTERISTICS OF THE MULTIPLE CHOICE QUESTIONS THAT HELP US TEST FOR THINKING

- The question has got more than one component to it
- The respondent is required to link different information
- The respondent needs to be able to analyze the scenario given to him
- Sometimes when the right answer has more than one component, then partial credit is possible
- The student can use different approaches to come up with the right answer
- The student needs to understand the link between different statistical concepts and methods
- The student needs to see the relationship between plots, equations, tables, etc.
- The student needs to be able to synthesize the information given to them to come up with the right answer
- The student needs to be familiar with the correct interpretation of the result

XI THE CHARACTERISTICS OF SHORT ANSWERS QUESTIONS THAT HELP US TEST FOR THINKING

- The question is short and precise
- The question does not take a long time to grade
- The students need to defend their answer
- The problem can have more than one right answer and so the students can use their creativity to come up with the right answer
- The students need to see the link between statistical concepts, plots, tables, equations, interpretations, etc.
- Possible correct answers are clear so that one can expect to have high inter-rater reliability between different graders

XII Sample multiple-choice questions

Question 1 (multiple choice): The following table shows the classification of a sample of 1300 students randomly selected from ten major Midwest universities by gender and major.

Gender	engineering	medicine	Social science
male	250	300	150
female	90	260	250

We want to see if field of study (major) depends on gender. What would be the best way for displaying this data?

- A bar chart with six bars; each bar representing the number of males and females within each major
- A bar chart with six bars; each bar representing the percentage of males and females within each field of study
- A segmented bar chart with two bars representing gender; with one bar showing the number of females within each field and the other bar showing the number of males within each field.
- A segmented bar chart with three bars representing the three fields of study, with each of the three bars showing the percentage of males and females within that field.

In order to get the right answer, the student needs to:

- Have a thorough understanding of the contingency table, independence, and the segmented bar chart
- Link the contingency table with the segmented bar chart

Question 2 (multiple choice): In a certain company the following information is available on the salary of male and female

employees.

For the male group more than 70% of the salaries are below the mean of the salary for the male employees. The data is unimodal.

For the female group more than 70% of the salaries are above the mean of the salary for the female employees. The data is unimodal.

How do you expect the side-by-side box plots for the salary of the males and females to look? Select the best answer.

- a) Both box plots will be skewed and the median line will not be in the middle of any of the boxes
- b) Both boxplots will be skewed, in the case of males the median line will be close to the bottom of the box and for the females the median line will be closer to the top of the box
- c) Both boxplots will be skewed, in the case of males the median line will be close to the top of the box and for the females the median line will be closer to the bottom of the box
- d) Need to have the actual data to compare the shape of the boxplots

In order to answer this question correctly, the student needs to

- Understand the concept of skewness
- Draw the histogram for both male and female and decide on the relative position of the median and the mean
- Given the histogram decide how the relevant boxplots will look like, thus needs to see the link between a boxplot and a histogram

Question 3 (multiple choice):

The average blood pressure for adults in the 30-40 year old age range is about 135. A researcher wonders whether the blood pressure of individuals with high stress positions differs from 135, keeping age range constant.

For a random sample of 120 people with high stress jobs, he found:

$\bar{X} = 137$

P value = 0.11

95% CI (139 to 141).

Based on these findings, he decided not to reject the null. Did he make the right decision?

What is the best answer?

- a) Yes, he made the right decision because the probability of rejecting the true null is 11%, and it is higher than the 5%.
- b) He did not make the right decision because his confidence interval does not include the hypothesized value under the null.
- c) The results reported by this researcher are inconsistent and contradictory.
- d) This researcher needs to report the standard deviation for his data.

In order to get the right answer, the student should

- Understand hypothesis testing and the meaning of the P value,
- Understand the meaning of the confidence interval, and
- Be able to link hypothesis testing and confidence interval and understand that they should give you consistent conclusions.

XIII SAMPLE SHORT-ANSWER QUESTIONS

Question 4; (short answer)

You are given two sets of data with the following information

	Data Set A	Ata Set B
Sample size	30	30
Mean	20	20
$\Sigma(X - \bar{x}) = 0$	0	0
Number of outlier*	3	3
Variance	0	225

The outliers for both data sets are two standard deviations below the mean.

Are the above results possible? If yes, explain why and if no explain why. A simple “yes” or “no” answer will not get any credit.

In order to get the right answer, the students should:

- Understand the meaning of variance and understand the impact of outliers on variance.
- The student can get the right answer by looking at the equation for the variance, understanding the verbal definition of variance, or drawing a histogram in which variance is zero.

Question 5 (Short answer)

Suppose that you are the teaching assistant for an introductory statistics course. A student asks you if the following distribution is normal or not? Will you tell him “Yes” or “No”? If yes why and if no why? A simple yes or no answer does not get any credit.

Mean = 55, SD = 8

Interval for X	Frequency in the interval
90-100	1
80-90	1
70-80	1
60-70	3
50-60	90
40-50	3
30-40	1
20-30	1
10-20	1

- In order to answer this question correctly, the student should understand the concept of symmetry, spread, and kurtosis.
- The student could get the right answer by finding out if the area within one and two standard deviations of the given fits the 68%, 95% rule.
- The student could get the right answer by visual inspection of the frequency table.
- The student could get the right answer by drawing the histogram of the data.

XIV How to design questions with partial credit that test for thinking

Present the students with a dilemma or a situation with real context and the relevant plot, numbers, printouts, etc.

Ask the students an open-ended questions to correct answer to which involves at least two components

Present the student with a number of alternative answers to the question such that

- One of the alternatives is completely wrong,
- One of the alternatives is completely right, and
- Several alternatives are partially right

XV Generic examples of questions with partial credit

Suppose you pose a question the answer to which involves two correct components. In that sense the alternative answers would be:

- Right, Right (two points)
- Wrong, Wong (zero points)
- Right, Wrong (one point)
- Wrong, Right (one point)

Suppose that you ask a question the answer to which involves three correct components, in that sense the alternative answers would be:

- Right, Right, Right (3 points)
- Wrong, Wrong, Wrong (zero points)
- Three possible combinations of Right, Right, Wrong (two points)
- Three possible combinations of Wrong, Wrong, Right (one point)

XVI SAMPLE OF QUESTIONS WITH PARTIAL CREDIT

Question 6: The objective of a study was to examine whether the average SATQ scores of the students who were admitted to a particular university was higher than the general population. Suppose that you were hired to analyze the relevant data and report the results to the registrar's office. Which of the following options would you pick to explain the findings?

$$\mu = 500$$

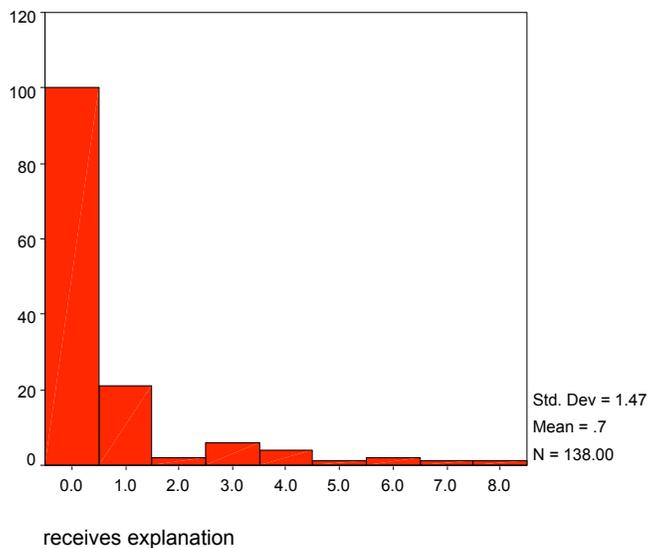
$$\sigma = 100$$

$$N = 225$$

$$\bar{x} = 523$$

- a) Since the risk with rejecting the true null is more than 5%, we fail to reject the null and conclude that SAT scores for the students in this university is similar to the general population (W/W = 0)
- b) Since the confidence interval does not include the hypothesized value under the null, we reject the null and conclude that the SAT scores for the students in this university should not be compared to the general population. (R/W = 1)
- c) On the average the students who are admitted to this university score 23 points higher than the general population on SATQ, but, this difference is not statistically significant (R/W = 1)
- d) We reject the null and we are 95% confident that the students who are admitted to this university score between 10 points to 36 points higher than the general population on SATQ.

Question 7. A math teacher uses cooperative groups to teach her class. You are given the histogram and the frequency data for the percentage of explanations received by students from their team mates during group work.



receives explanation					
		Frequency	Percent	Valid Percent	Cumulativ Percent
Valid	0	100	72.5	72.5	72.
	1	21	15.2	15.2	87.
	2	2	1.4	1.4	89.
	3	6	4.3	4.3	93.
	4	4	2.9	2.9	96.
	5	1	.7	.7	97.
	6	2	1.4	1.4	98.
	7	1	.7	.7	99.
	8	1	.7	.7	100.
	Total	138	100.0	100.0	

Given the above plots, what is the best conclusion?

- a) The graph is negatively skewed and the majority of students (more than 70%) are receiving explanations during group work. (Wrong, Wrong = 0).
- b) The graph is positively skewed and the majority of the students (more than 70%) are receiving explanations during group work (Right, Wrong = 1).
- c) The graph is negatively skewed and the mean is less than the median of the explanations received during group work. (Wrong, Wrong = 0)
- d) The graph is positively skewed and the majority of the students (more than 70%) are not receiving explanations during group work. (Right, Right = 1)

